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Restrictions on dilatonic brane-world models

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Abstract

We consider dilatonic brane-world models with a non-minimal coupling between a dilaton and usual matter on a brane. We demonstrate that variation of the fundamental constants on the brane due to such interaction leads to strong restrictions on parameters of models. In particular, the experimental bounds on the variation of the fine structure constant rule out non-minimal dilatonic models with a Liouville-type coupling potential $f(\varphi) = \exp(b\varphi)$ where $b \sim \mathcal{O}(1)$.

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Brane-world models have been the subject of intensive investigation for the last few years. They offer an interesting alternative (with respect to the Kaluza-Klein model) to the standard multidimensional gravity and cosmology. The main feature of this approach consists in a proposal where the standard matter (SM) fields are localized on the brane (4-dimensional hypersurface which corresponds to our Universe) whereas the gravitational field can propagate in the full multidimensional space-time. It sheds a new light on the problem of the large hierarchy and leads to new designing properties and phenomena for multidimensional models. Thus, it is important to predict observable effects which can confirm such brane-world approach.

Obviously, SM particles may escape from the brane into a bulk resulting in the violation of the energy-momentum and charge conservation laws in the brane [1]. Such effect can take place, for example, if SM particles interact with bulk fields. A lot of papers were

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devoted to the problem of the interaction between radions and SM fields (see [2], [3] and references therein). Radions usually describe relative motion of the branes. For realistic models, it is usually supposed that there is a mechanism for the brane stabilization with respect to each other. Let b_0 be the scale of stabilization of the inter-brane distance and $\psi(x)$ the small fluctuations (radions) around it. Then, an induced 4-D metric on the brane located in an additional dimension at $y = y_0$ reads: $h_{\mu\nu}(x, y_0) = A_0 \exp(c_0\psi(x))\tilde{h}_{\mu\nu}(x)$, where A_0 is a dimensionless warp factor corresponding to the scale of stabilization b_0 and $c_0 \sim 1/M_{EW}$ (in the ADD (Arkani-Hamed, Dimopoulos, Dvali) brane approach $c_0 \sim 1/M_{Pl}$ [3], [4]). Let $\Phi(x)$ represents a matter field (SM) on the D_0-dimensional brane with a Lagrangian $L = L(\Phi(x), h(x, y_0))$ and following action

$$S = \int d^{D_0}x\sqrt{|h|}L(\Phi(x), h(x, y_0))\,. \tag{1}$$

The corresponding Lagrangian density of the interaction between radions and field Φ is

$$\mathcal{L}_{int} = \psi \frac{\delta \mathcal{L}}{\delta \psi}\bigg|_{\psi=0} = \psi\left(\frac{\delta \mathcal{L}}{\delta h^{\mu\nu}}\frac{\delta h^{\mu\nu}}{\delta \psi}\right)_{\psi=0} = -(c_0/2)\psi\left(\sqrt{|h|}\,T^{\mu}_{\mu}\right)_{\psi=0}\,, \tag{2}$$

where T^{μ}_{μ} is a trace of the energy-momentum tensor for the Lagrangian L with respect to the metric $h_{\mu\nu}$: $T_{\mu\nu} = -2\delta L/\delta h^{\mu\nu} + h_{\mu\nu}L \Leftrightarrow \sqrt{|h|}T_{\mu\nu} = 2\delta\mathcal{L}/\delta h^{\mu\nu}$, $\mathcal{L} = \sqrt{|h|}L$. Thus, the interaction between radions and SM field is absent for fields with vanishing T^{μ}_{μ}, e.g. for massless fermions and massless gauge bosons which are the quanta present in high-energy experiments. By this reason graviscalars were neglected in colliding experiments for studding of the brane-world physics[1].

Nevertheless, massless SM particles on the brane can interact at tree level with other bulk fields, e.g. with a non-minimal dilaton field. Moreover, as this scalar field lives in full 5-D space-time (in the bulk), the coupling constant is $c_0 \sim 1/M_{EW}$ (if 5-D gravitational constant $\kappa_5^2 = M_{EW}^{-3}$). Such interaction may play an important role in the brane-world physics. Thus, it is of interest to predict observable effects following from this type of interaction and to obtain experimental restrictions on parameters of the models. There is an extensive list of papers devoted to the investigation of the dilatonic brane-world models with a slightly different form of the action (e.g. [7] - [21]). They naturally follows from a low-energy limit of string theories and have a dilatonic bulk potential and a dilatonic coupling potential of the form of the Liouville potential [7], [8], [9], [14], [19]. In this paper, we choose the action in the general form

$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{|g|}\{R[g] - g^{ab}\partial_a\varphi\partial_b\varphi - 2\kappa_5^2 V(\varphi)\} + \tag{3}$$

[1] We should emphasize that in the standard Kaluza-Klein model the interaction (with $c_0 \sim 1/M_{Pl}$) between graviscalars (gravexcitons [5])and massless particles is possible at tree level [6].

$$+ \int_{M_4} d^4x\sqrt{|h|}\{-T + f(\varphi)L_m\} + \text{boundary term}\,,$$

where M_5 is the 5-D manifold with metric g_{ab} $(a, b = 0, 1, 2, 3, 5)$ and the 4-D hypersurface M_4 is the brane with induced metric[2] $h_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$. T is a tension of the brane and may also depend on dilaton φ. The Lagrangian L_m corresponds to SM fields on the brane. 5-D gravitational constant κ_5^2 is connected with 5-D fundamental mass as follows: $\kappa_5^2 = M^{-3}$ and we usually suppose $M = M_{EW} \sim 1TeV$. The dilatonic field φ is dimensionless. Its dimensions are restored with the help of the 5-D fundamental mass M: $\varphi = M^{-3/2}\bar{\phi} = M^{-1}\phi$ where $\bar{\phi}$ and ϕ have dimensions of $\mathcal{O}(m^{3/2})$ and $\mathcal{O}(m)$ (m is a unit of mass), respectively. A scalar field ϕ has usual dimensions for scalar fields in 4-D space-time (cf. [20]). The bulk potential $V(\varphi)$ has dimensions $\mathcal{O}(m^5)$, the brane coupling function $f(\varphi)$ is dimensionless and the brane tension T and the matter Lagrangian L_m have dimensions $\mathcal{O}(m^4)$.

It is clear that a non-minimal interaction of the dilatonic field with SM fields results in violation of the matter conservation on the brane (see footnote 3 below). To be in accordance with observations, φ should be stabilized on the brane near some value φ_0 or slightly vary during the Universe evolution (at least from the time of nucleosynthesis). Let φ_0 is the present value of φ and $\eta = M^{-1}\psi$ are small fluctuations around it. Then, the Lagrangian of interaction at tree level is

$$L_{int} = \eta \left. \frac{\delta\left(f(\varphi)L_m\right)}{\delta\varphi}\right|_{\varphi=\varphi_0} = \beta\frac{\psi}{M}L_m\,, \qquad (4)$$

with the coefficient $\beta := \delta f/\delta\varphi|_{\varphi_0}$. This formula shows that dilatonic fields can interact with massless SM particles at the tree level. It is the main difference with graviscalars considered in Eq. (2). Interaction (4) is suppressed by the electroweak mass $M = M_{EW} \sim 1TeV$ in contrast to the interaction with WIMP's (Weakly-Interacting Massive particles) which are suppressed by 4-D Planck mass $M_{Pl} \sim 10^{16}TeV$. Thus, the interaction SM fields with dilatons in brane worlds can be much more effective than with WIMP's in the standard Kaluza-Klein approach.

Obviously, interactions between dilatons and massless SM particles e.g. photons with $L_m = F_{\mu\nu}F^{\mu\nu}$ and massless fermions with $L_m = \bar{\Psi}\gamma^\mu\partial_\mu\Psi$ are of the most interest in high-energy colliding experiments. If the dilaton field φ is stabilized on the brane at φ_0 corresponding to a minimum of an effective potential and small fluctuations near this

[2] For simplicity, we consider the case of one brane located at the additional coordinate $y = y_0$. Let n_a be a unite space-like vector normal to the brane. Then, the induced metric on the brane is $h_{ab} = g_{ab} - n_a n_b$. We also suppose that all space-time can be covered by the normal Gauss coordinates where $n_a = n^a = (0, 0, 0, 0, 1)$. In this case $h_{a5} = h^a_5 = h^5_a = 0$ and $h^\mu_\nu = \delta^\mu_\nu$. These simplifications do not affect the results of our paper.

position constitute quanta ψ with a mass m, then a decay rate of these quanta into 2 photons or 2 massless fermions are

$$\Gamma \sim \beta^2 \frac{m^3}{M^2} \tag{5}$$

with a life-time $t \sim 1/\Gamma \sim \beta^{-2}(M^2/m^3)(\hbar/c^2)$. Thus, the dilatons with masses

$$m \lesssim \beta^{-2/3} \left[\frac{T_{Pl}}{t_{univ}} M^2 M_{Pl} \right]^{1/3} \sim \beta^{-2/3} 10^{-4} \mathrm{eV} \tag{6}$$

have life-time $t \geq 10^{19}$sec $> t_{univ} \sim 10^{18}$sec greater than the age of the Universe. They are rather light particles. For heavier dilatons the decay plays important role during the Universe evolution.

It is well known (see e.g. [22]) that interaction of the form $f(\varphi)F^2$ results in variation of the fine structure constant α :

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{f}}{f}, \tag{7}$$

where the dot denotes differentiation with respect to time. There is an extensive list of papers devoted to the experimental bounds for such variations (e.g. [23], [24] and references therein). Different experiments give different bounds on $|\dot{\alpha}/\alpha|$, from $\lesssim 10^{-12}\mathrm{yr}^{-1}$ for cosmic microwave background [23] to $\lesssim 10^{-17}\mathrm{yr}^{-1}$ for the Oklo experiment [25]. Primordial nucleosynthesis gives $|\Delta\alpha/\alpha| \lesssim 10^{-4}$ at a redshift on the orders $z = 10^9 - 10^{10}$ [26], i.e. $|\dot{\alpha}/\alpha| \lesssim 10^{-14}\mathrm{yr}^{-1}$. In all these estimates $\dot{\alpha} = \Delta\alpha/\Delta t$ is the average rate of change of α for the period Δt (corresponding to a redshift z). For our calculations we take some averaged estimate $|\dot{\alpha}/\alpha| \lesssim 10^{-13}\mathrm{yr}^{-1}$ which corresponds to a Hubble time scale $\Delta t \sim H_0^{-1} \sim 10^{10}$ years. For this bound, from Eq. (7) we obtain:

$$\left| \frac{\dot{f}}{f} \right| = \left| \frac{\dot{\phi}}{M} \frac{1}{f} \frac{df}{d\varphi} \right| \lesssim 10^{-13}\mathrm{yr}^{-1}, \tag{8}$$

This estimate leads to the following restriction on the parameter β (cf. [22]):

$$|\beta| \approx \Delta t \left| \frac{\dot{\alpha}}{\alpha} \right| \frac{M}{\Delta\phi} \;\Rightarrow\; |\beta| \lesssim 10^{-3}, \tag{9}$$

where we suppose $\Delta\phi \sim M$ and that the present value of $f \approx 1$ (that usualy is equivalent to the assumption for the dilaton field at the present time: $\phi_0 \ll M \Rightarrow \varphi_0 \ll 1$).

As we wrote above, most of the dilatonic models are motivated by string theories which, at a low-energy limit, usually have the Liouville-type potentials: $V(\varphi) = V_0 \exp(a\varphi)$ and $f(\varphi) = \exp(b\varphi)$ with $a = 2b = \mathcal{O}(1)$ [8], [9], [14], [19]. Substitution of the Liouville coupling potential $f(\varphi) = \exp(b\varphi)$ into estimate (8) leads to the limits on the parameter b:

$$\left| b \frac{1}{M} \frac{\Delta\phi}{\Delta t} \right| \lesssim 10^{-13}\mathrm{yr}^{-1} \;\Rightarrow\; |b| \lesssim 10^{-3} \tag{10}$$

for $\Delta\phi \sim M$ and $\Delta t \sim 10^{10}$ years. Estimates (9) and (10) coincide with each other because for $\varphi_0 \ll 1 : \beta = \delta f/\delta\varphi|_{\varphi_0} = b\exp(b\varphi_0) \sim b$.

It is hardly possible that the Liouville-type potentials for such considered model provide the stabilization of φ on the brane (see Eq. (18) below). Thus, the dilatonic models with non-minimal coupling to the SM fields on the brane are ruled out by estimate (10) for theories with $a \sim b \sim \mathcal{O}(1)$.

Another restrictions on parameter of models can be obtained from experiments on variation of the 4-D gravitational constant. It is well known (see e.g. [16] and [17]) that in 4-D projected Einstein equation the term linear with respect to the brane matter energy-momentum[3] $\tau_{(m)\mu\nu} = -2\delta L_m/\delta h^{\mu\nu} + h_{\mu\nu}L_m$ has the form $(\kappa_5^4/6)Tf(\varphi)\tau_{(m)\mu\nu}$, and namely this term is responsible for the conventional cosmology on the brane. Thus, the quantity

$$8\pi G_N \equiv \frac{\kappa_5^4}{6}Tf(\varphi) \tag{11}$$

plays the role of a 4-D gravitational constant on the brane[4]. This equation shows that effective 4-D gravitational constant G_N for models (3) depends on the function $f(\varphi)$ (for simplicity, we impose that the tension $T \equiv$ const). Thus, variation of $f(\varphi)$ leads to a variation of G_N. There are a number of observable data for an estimate of a possible time variation of the gravitational constant [27], [28]. They imply $|\dot{G}_N/G_N| \lesssim 10^{-11}\text{yr}^{-1}$. Thus, we can obtain a limitation of the variation of $f(\varphi)$:

$$\left|\frac{\dot{f}}{f}\right| \lesssim 10^{-11}\text{yr}^{-1}\,, \tag{12}$$

which for the Liouville potential $f = \exp(b\varphi)$ puts on the parameter b the following restrictions:

$$|b| \lesssim 10^{-1}\,, \tag{13}$$

where we suppose $\Delta\phi/\Delta t \sim M/10^{10}\text{yr}$. This estimate is much less severe than (10) and, strictly speaking, not rules out theories with $b \sim \mathcal{O}(1)$. For example, it is expected [28], that on the Hubble time scale $|\dot{G}_N/G_N| \lesssim H_0 \sim 10^{-10}\text{yr}^{-1}$. Then, inequality (13) is reduced to the following estimate: $|b| \lesssim 1$.

In order to avoid the problem of the fundamental constant variation in the dilatonic non-minimal models, it is natural to suppose that the dilaton is stabilized on the brane

[3]An effective energy-momentum conservation equation for the matter on the brane has the form [17] $\bar{\nabla}^\nu(f(\varphi)\tau_{(m)\mu\nu}) = (\bar{\nabla}_\mu\varphi)(df/d\varphi)L_m$, which shows that the matter is conserved on the brane if the dilaton field is either minimally coupled to the SM matter ($f \equiv$ const) or stabilized on the brane ($\varphi|_{brane} \to$ const).

[4]This was the main reason for us to include the tension term in action (3), because, if $T \equiv 0$, then the linear τ-term is absent in the projected Einstein equation.

(before primordial nucleosynthesis), i.e. $\varphi \to \varphi_0 \equiv \text{const}$ where φ_0 corresponds to a stable solution of the equation of motion on the brane.

The equation of motion for φ reads

$$\Box[g]\varphi = \kappa_5^2 \frac{dV}{d\varphi} + \frac{\sqrt{|h|}}{\sqrt{|g|}}\delta(y-y_0)\kappa_5^2 \left(\frac{dT}{d\varphi} - \frac{df}{d\varphi}L_m\right) , \tag{14}$$

where $\Box[g] = g^{ab}\nabla_a\nabla_b$ and $\nabla_a \equiv \nabla_a[g]$ is the covariant derivative with respect to 5-D metric g_{ab}. In the normal Gauss coordinates $|h| = |g|$. The jump condition (at the brane) corresponding to Eq. (14) is (see also [7], [17] and [21])

$$\begin{aligned} [n^a\partial_a\varphi]_-^+ &= \kappa_5^2 \left(\frac{dT}{d\varphi} - \frac{df}{d\varphi}L_m\right)_{brane} \\ \Rightarrow\ n^a\partial_a\varphi|_{brane} &= \partial_y\varphi|_{brane} = \frac{1}{2}\kappa_5^2 \left(\frac{dT}{d\varphi} - \frac{df}{d\varphi}L_m\right)_{brane} , \end{aligned} \tag{15}$$

where in the latter equality we use the normal Gauss coordinates and impose $\mathbb{Z}_2$ symmetry. The unite vector n usually points into the bulk. The last relation in (15) is used for evaluation of $\partial_y\varphi$ close to the brane. Thus, after some algebra (see e.g. [17]) the 4-D projected equation for scalar field φ on the brane reads

$$\Box[h]\varphi + \frac{\partial^2\varphi}{\partial y^2} = \kappa_5^2\frac{dV}{d\varphi} - \frac{2}{3}\kappa_5^4 T\frac{df}{d\varphi}L_m - \delta(y-y_0)\kappa_5^2\frac{df}{d\varphi}L_m\, , \tag{16}$$

where $\Box[h] = h^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu$ and $\bar{\nabla}_\mu \equiv \bar{\nabla}_\mu[h]$ is the covariant derivative with respect to 4-D metric $h_{\mu\nu}$ and we impose that the tension T does not depend on the scalar field φ. The δ-function in this equation is canceled by second derivative $\partial^2\varphi/\partial y^2$, resulting in jump condition (15).

As follows from Eq. (16), to define the behaviour of φ on the brane, we should obtain first a solution for the bulk Eq. (14) with an appropriate boundary conditions. One of them is the jump condition (15) and another one is defined by the topology of the model and the form of the potential $V(\varphi)$. Following paper [16], we can expand the dilaton field φ near the brane as

$$\varphi(x,y) = \tilde{\varphi}(x) + \Phi_1(x)|y| + \frac{1}{2}\Phi_2(x)y^2 + \mathcal{O}(y^3)\, . \tag{17}$$

Inserting this expansion into Eq. (16), we find that Φ_1 satisfies the jump condition (15): $\Phi_1 = -(1/2)\kappa_5^2\,(df/d\varphi)\,L_m$ and for $\tilde{\varphi}$ we obtain equation

$$\Box[h]\tilde{\varphi}(x) = \kappa_5^2\frac{dV}{d\varphi} - \frac{2}{3}\kappa_5^4 T\frac{df}{d\varphi}L_m - \Phi_2(x)\, . \tag{18}$$

Here, $\Phi_2(x)$ is defined from a solution of the bulk Eq. (14) with an appropriate boundary conditions. Thus, the main problem of the non-minimal dilatonic brane-world models consists in the construction of models where $\tilde{\varphi}_0 \equiv$ const is a stable solution of Eq. (18). If, in general, such constructions are impossible, then variations of φ with time should be in accordance with experimental bounds on variations of the fundamental constants (see e.g. Eqs. (8) and (12)).

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